VII-1 MICROWAVE PROPAGATION AND FARADAY EFFECT IN A SOLID STATE PLASMA WAVEGUIDE

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The conductivity of a solid-state plasma which consists of free electrons and holes in a solid such as a semiconductor becomes a tensor quantity under a dc magnetic field. If we take a coordinate system in such a way that the dc magnetic field B_o lies along the z-axis, the tensor conductivity may be expressed by

$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_\perp, & -\sigma_x, & 0 \\ \sigma_x, & \sigma_\perp, & 0 \\ 0, & 0, & \sigma_{//} \end{bmatrix}$$

where if we assume that $n_e\mu_e \gg n_h\mu_h$ as in the case of an n-type semiconductor, we have

$\sigma_{//} = qn_e\mu_e/(1+j\omega\tau_e)$, $\sigma_\perp = \sigma_{//}/(1+a_e^2)$, $\sigma_x = \sigma_{//}a_e/(1+a_e^2)$, and $a_e = \mu_e B_o/(1+j\omega\tau_e)$. ω is the angular frequency, q the electron charge, n the density, μ the mobility, and τ the relaxation time. The subscripts e and h refer to electrons and holes, respectively. Variations of $\sigma_{//}$, $\sigma_\perp$, and σ_x with B_o are shown in Fig. 1.

Propagation of electromagnetic waves in such an anisotropic medium is very complex and interesting. In particular, when a linearly polarized electromagnetic wave propagating along the magnetic field enters the anisotropic medium, the plane of polarization experiences a rotation. In this paper theoretical analysis and experimental investigation of microwave propagation and the Faraday effect in a circular solid-state plasm waveguide (rather than in an unbound infinite space) under a longitudinal magnetic field are presented. Possible microwave device applications of the Faraday effect in a solid-state plasma at millimeter and sub-millimeter wave regions are also discussed and experimentally demonstrated.

The field configurations and propagation characteristics in the solid-state plasma waveguide are first analyzed by solving the Maxwell equations together with the equation of motion for given boundary conditions. The analysis shows that there exist nondegenerate right-hand and left-hand circularly polarized quasi-TE modes in the solid-state plasma waveguide. By means of a variational method, the propagation constant and characteristic wave impedance of each mode that exist in the waveguide are then obtained. The propagation constants of the two types of circularly polarized quasi-TE waves in the solid-state plasma waveguide are given by

$$k_{\pm nm}^2 = \omega^2\mu_o\epsilon - k_{Tnm}^2 - j\omega\mu_o\sigma_\perp \pm \omega\mu_o\sigma_x I_{nm}$$

where μ_o is the free-space permeability, ϵ the dielectric constant of the material. The subscripts + and − refer to the left-hand and right-hand circular polarizations, respectively. The subscript nm indicates the order of modes as in the case of TE waves in ordinary waveguides. The constant k_{Tnm} and I_{nm} are tabulated for various modes in Table I. The two types of circularly polarized quasi-TE waves can propagate with very small attenuation in the solid-state plasma waveguide under a relatively high magnetic field, i.e., $\mu_e B_o \gg 1$.

A linearly polarized TE wave in an empty circular guide can be considered to consist of degenerate right-hand and left-hand circularly polarized waves of equal amplitude. In the solid-state plasma, however, the two types of circularly polarized waves are nondegenerate, i.e., they have different propagation constants and different characteristic wave impedances. Therefore, in a waveguide as shown in Fig. 2, the right-hand and left-hand circularly polarized waves have different transmission coefficients. The transmission coefficients of the two circularly polarized waves are given by

$$T_{\pm nm} = |T_{\pm nm}| \, e^{i\psi_{\pm nm}} = \frac{(1 - \Gamma^2_{\pm nm})e^{-ik_{\pm nm}\ell}}{(1 - \Gamma^2_{\pm nm}e^{-i2k_{\pm nm}\ell})}$$

where

$$\Gamma_{\pm nm} = \frac{Z_{onm} - Z_{\pm nm}}{Z_{onm} + Z_{\pm nm}} = \frac{k_{\pm nm} - k_{Tnm}}{k_{\pm nm} + k_{Tnm}}$$

The polarization of the wave transmitted through the solid-state plasma waveguide are in general elliptically polarized. The major axis of polarization of the wave is rotated with respect to the polarization of the incident wave through an angle of

$$\theta = (\psi_+ - \psi_-)/2$$

The ellipticity of polarization $\mathcal{E}$ is given by

$$\mathcal{E} = (|T_+| - |T_-|)/(|T_+| + |T_-|)$$

The transmission coefficient and the insertion loss of the solid-state plasma waveguide are given by

$$T = (|T_+| + |T_-|)/2$$

$$L = -20 \log T$$

It is shown that the reflections from the waveguide discontinuities have significant effect on the angle of the Faraday rotation and the ellipticity of polarization of the transmitted waves in the solid-state plasma waveguide.

Experimental investigation of the Faraday effect in a solid-state plasma waveguide is also reported. In the experiment, n-type indium antimonide single crystals are used since they have high electron mobilities. The experiment is conducted with the frequency about 36 KMC and at liquid nitrogen temperature. The angle of the Faraday rotation, the ellipticity, and the insertion loss through circular discs of 0.5mm and 1.0mm thickness which are mounted in a circular waveguide as shown in Fig. 2 are measured under a longitudinal magnetic field up to 14 KG. By placing quarter-wavelength impedance transformers at both sides of the indium antimonide disc, the reflection coefficient is changed. The influence of the reflections from the waveguide discontinuities on the Faraday effect is thus examined. The comparison of the experimental results and the theoretical analysis shows very good agreement as shown in Fig's. 3, 4, and 5.

An experimental solid-state plasma nonreciprocal microwave device that makes use of the large Faraday rotation with very small attenuation was developed and constructed. The nonreciprocal transmission characteristics of the experimental device are then investigated. The isolation ratio of as much as 30 dB was obtained by the experimental device. The theoretical upper frequency limit of the solid-state plasma when used as microwave devices appears at the cyclotron resonant frequency which is higher than 1000 KMC in the present case. It is pointed out, therefore, that the solid-state plasma may find important applications in the development of nonreciprocal microwave devices in the millimeter and submillimeter wave regions.

TABLE I – NUMERICAL VALUE OF K_{Tnm} AND I_{nm}

Values of $K_{Tnm}r_o$

m	n	0	1	2	3	4	5
1		3.832	1.841	3.054	4.201	5.307	6.416
2		7.016	5.331	6.706	8.015	9.282	10.520
3		10.173	8.536	9.969	11.346	12.682	13.987
4		13.324	11.706	13.170			

Values of I_{nm}

m	n	0	1	2	3	4	5
1		0	0.837	0.753	0.703	0.654	0.411
2		0	0.700	0.198	0.108	0.102	0.100
3		0	0.028	0.042	0.049	0.052	0.054
4		0	0.0015	0.024			

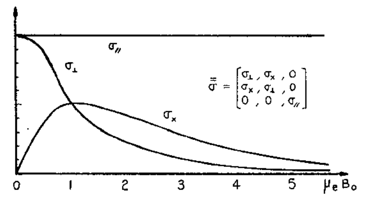

FIG. 1 - Variation of the tensor conductivity of a solid-state plasma as a function of applied magnetic field.

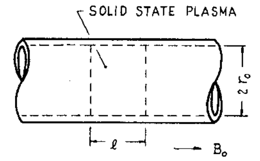

FIG. 2 - A circular solid-state plasma waveguide

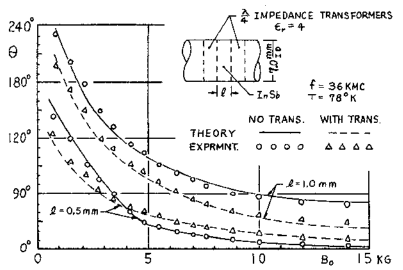

FIG. 3 - Calculated and measured angle of Faraday rotation in the solid-state plasma waveguide as a function of the magnetic field

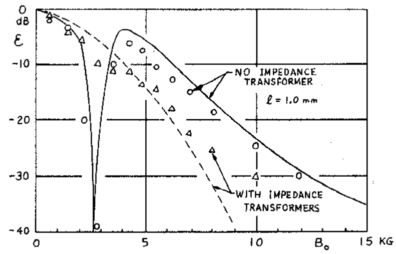

FIG. 4 - Calculated and measured ellipticity of polarization of the transmitted wave in the solid-state plasma waveguide as a function of the magnetic field

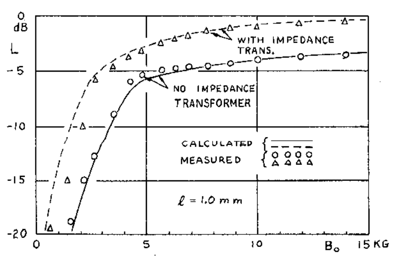

FIG. 5 - Calculated and measured insertion loss of the solid-state plasma waveguide as a function of the magnetic field..

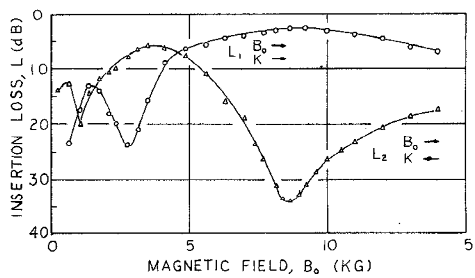

FIG. 6 - Nonreciprocal transmission characteristics of the experimental solid-state plasma microwave device